

AVERAGE RANK AND AVERAGE ROOT NUMBER OF ELLIPTIC CURVES

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In this talk, we will be interested in elliptic curves, and the group structure on the points of elliptic curves. If one considers the points on an elliptic curve with rational coordinates, those points form a finitely generated abelian group, and the rank of the free part of this group is called the rank of the elliptic curve. Though easy to define, the rank of elliptic curves is still an elusive object to mathematicians, and many questions are still open : Is the rank of elliptic curves bounded ? What is the average size of the rank as one varies over families of elliptic curves ?

The rank of elliptic curves is also related to the order of vanishing of some analytic function associated to the elliptic curve by the famous conjecture of Birch and Swinnerton-Dyer, one of the 7 Clay Millenium Problems. We will explain this conjecture, and use this connection to define the root number of elliptic curves, and present some families of elliptic curves where the behaviour of the root number, and then of the rank, is unexpected.